Optimal boundary control for steady Navier-Stokes equations with directional do-nothing boundary condition

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Talk Abstract

Inspired by the modelling of blood flow redirected by a bypass surgery in a tract of an artery (see [1]), we study the Navier-Stokes equations with mixed boundary conditions. As in [1], a non-homogeneous Dirichlet boundary condition is considered in the inlets of the fluid domain. In our work, instead of the classical do-nothing condition, we impose a directional do-nothing (DDN) outflow boundary condition (see [2, 3, 4, 5]).

Based on a saddle point approach (see [1]), we begin by establishing the well-posedness of the direct problem. Then, aiming at flow regularization, we analyse the boundary control problem which consists in the minimization of quadratic cost functionals of the velocity field (tracking-type or vorticity) by means of the inflow velocity in one inlet. We prove the existence of optimal solutions, justify the Gâteaux derivative of the control-to-state map and deduce the first order necessary conditions for optimality. The results are obtained under smallness restrictions on the inflow boundary controls.

Keywords: Navier-Stokes equations, mixed boundary conditions, directional do-nothing condition, optimal boundary control.

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