Limiting cases in Choquard type equations

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Talk Abstract

We will present recent results for a class of Choquard type equations of the following form

$$(C) \quad -\Delta_N u + V(x)u = (I_N * F(x,u))f(x,u), \quad x \in \mathbf{R}^N, N \geq 2$$

where I_N is the Riesz logarithmic kernel, V is a bounded Schrödinger potential and the nonlinearity $f(x,u)$, whose primitive in u vanishing at zero is $F(x,u)$, exhibits the highest possible growth – for functions with membership in $W^{1,N}(\mathbf{R}^N)$ – which is of exponential type. The competition between the logarithmic kernel and the exponential nonlinearity demands for new tools. A proper function space setting is provided by a new weighted version of the Pohozaev–Trudinger inequality which enables us to prove the existence of variational, in particular finite energy solutions to (C). Equivalence issues with connected higher order fractional Scrödinger-Poisson systems will be also discussed, as well as related open problems.

Keywords: Higher order fractional Schrödinger-Poisson systems, Schrödinger-Newton equations, Weighted Trudinger-Moser type inequalities in $\mathbf{R}^N$, Variational methods.

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