Quadratic PDE and optimal ballistic problems

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Talk Abstract

Optimal ballistic problems are somewhat similar to the shortest path (geodesic) problems. In the finite-dimensional setting, say, on a Lie group with a Riemannian metric, we minimize the kinetic energy along all curves with presribed final point and initial velocity. Remarkably, such problems naturally arise in the infinite-dimensional setting from a certain dual formulation of nonlinear evolutionary PDE, cf. [1, 2]. The examples include the incompressible Euler, inviscid Burgers, ideal incompressible MHD, the template matching equation, the multidimensional Camassa-Holm (also known as the H(div) geodesic equation), EPDiff, Euler-alpha, KdV and Zakharov-Kuznetsov equations, the equations of motion for the incompressible isotropic elastic fluid and for the damping-free Maxwell's fluid. This yields the existence of a new type of generalized solutions to the initial-value problems for the above mentioned PDE. We also discuss a sharp upper bound on the optimal value of the dual problem and the weak-strong uniqueness issue.

Keywords: optimal transport, fluid dynamics, hidden convexity, Euler-Arnold equations.

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