Existence and non-existence of positive solutions for the critical p-Laplace equation in the ball

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Talk Abstract

We focus on positive radial solutions to the Dirichlet problem associated with the generalized scalar curvature equation

$$\begin{cases} \Delta_p u + K(|x|)u^{q-1} = 0, & x \in B_R(0) \\ u(x) = 0 & |x| = R, \end{cases}$$

where $\Delta_p u = \operatorname{div}(\nabla u |\nabla u|^{p-2})$ denotes the p-Laplace operator, $B_R(0)$ is the ball of radius $R > 0$ in $\mathbb{R}^n$, $2n/(2+n) \leq p \leq 2$, $n > p > 1$ and q is the Sobolev critical exponent

$$q = p^* = \frac{np}{n-p}.$$

The function K is assumed to be C^1, bounded, positive and to satisfy the ℓ-flatness condition. In particular, we show that the existence of positive solutions depends on the slope ℓ of K at zero, and on the length of the radius R. Our main purpose is to improve and extend the result in [1] to the p-Laplacian case. Interesting results can also be achieved under an additional monotonicity assumption on K. Our approach, based on Fowler transformation, invariant manifold theory, phase plane analysis, and energy estimates, offers a new geometrical perspective and exploits the construction of suitable barrier sets for the solutions.

Keywords: Scalar curvature equation, Fowler transformation, Invariant manifold, Phase plane analysis.

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References

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