Periodic solutions to the relativistic Kepler problem: a dynamical systems approach

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Talk Abstract

The motion of a relativistic particle in a Kepler potential can be described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3}, \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $m > 0$ is the mass of the particle, c is the speed of light, and $\alpha > 0$ is a constant. Firstly, we illustrate the Hamiltonian formulation of the problem and we focus our attention on the description of the periodic and quasi-periodic solutions. Secondly, we deal with the perturbed equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3} + \varepsilon\,\nabla_x U(t,x), \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $U(t,x)$ is T-periodic in the first variable and $\varepsilon \in \mathbb{R}$. The analysis of the action-angle coordinates and an application of an higher dimensional version of the Poincaré–Birkhoff fixed point theorem allow to prove that, for ε small enough, the perturbed problem admits T-periodic solutions with prescribed winding number, bifurcating from invariant tori of the unperturbed problem. The talk is based on the paper [1] written in collaboration with Alberto Boscaggin and Walter Dambrosio.

Keywords: relativistic Kepler problem, periodic solutions, invariant tori, nearly integrable Hamiltonian systems, action-angle coordinates.

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References

[1] Boscaggin, A., Dambrosio, W. and Feltrin, G., Periodic solutions to a perturbed relativistic Kepler problem, *SIAM J. Math. Anal.*, 53(5), 2021, pp. 5813–5834.