Unbounded spectral minimal partitions?

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Talk Abstract

Spectral minimal partitions (SMPs) of bounded domains in Euclidean space have been studied intensively over the last two decades, ever since the pioneering work of Conti, Terracini and Verzini (e.g., [1]; see also [2] for a relatively recent survey). In general, one seeks to minimise, among all suitable partitions of the domain, a functional built on Laplacian eigenvalues of the partition pieces; the solution can also typically be attained as a limit solution to competing species-type elliptic systems as the competition term becomes stronger. Beyond their links to competing systems, SMPs are studied both due to their links to spectral properties of the Laplacian on the whole domain, and the problem of dividing a domain optimally into a given number of pieces which are "equal" in some analytic sense. Corresponding problems on graphs are closely related to the search for clusters in the graphs. We will explore the problem of partitioning *unbounded* domains $\Omega \subseteq \mathbb{R}^n$ of infinite volume in Euclidean space, equipped with a nonnegative potential $q : \Omega \to \mathbb{R}$ acting as a "landscape" on the domain. We will formulate a number of conjectures relating the existence, or non-existence, of SMPs of Ω to the infimum of the essential spectrum of the Schrödinger operator $-\Delta + q$ on Ω. This is in large part based on ongoing joint work with Matthias Hofmann (Texas A&M University) and Andrea Serio (Faculdade de Ciências da Universidade de Lisboa), where we prove analogous results in the setting of quantum graphs.

Keywords: Schrödinger operator, eigenvalues, quantum graph, unbounded domain.

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References

[1] Conti, M., Terracini, S. and Verzini, G., On a class of optimal partition problems related to the Fučík spectrum and to the monotonicity formulae, *Calc. Var. Partial Differential Equations*, 22(1), 2005, pp. 45–72.

[2] Bonnaillie-Noël, V. and Helffer, B., Nodal and spectral minimal partitions—the state of the art in 2016, pp. 353–397 in Henrot, A. (ed.), *Shape optimization and spectral theory*, De Gruyter Open, Warsaw, 2017.