Existence of solutions to the Navier-Stokes Cauchy problem in the L^3 setting

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Talk Abstract

We investigate on the existence of solutions to the Navier-Stokes Cauchy problem with initial datum u_0 in L^3 and divergence free. It is known that this kind of result is not new. Indeed, there is a wide literature on it, with a first contribution due to T. Kato in [4]. Our chief goal is to establish the existence interval $(0, T)$ by uniquely considering the size of the initial datum in L^3 and the absolute continuity of $|u_0(x)|^3$.

A similar analysis has been developed in the recent paper [1], where it is employed the dimensionless weighted functional $||U_0||^2_{wt} := \sup_x \int_{\mathbb{R}^3} \frac{U_0^2(y)}{|x-y|} dy$ and, in the set L^2_{wt}, where $||\cdot||_{wt} < \infty$, the subset of the so called Kato class K_3 is considered. In this regard, we recall that $||\cdot||_{wt}$ is not equivalent to the L^3-norm.

The result, proved in paper [2] for the Cauchy problem, will represent the starting point for the same result, in a forthcoming paper [3], in the case of the initial boundary value problem in $(0, T) \times \Omega$, where $\partial\Omega$ is assumed a sufficiently regular compact set, or is the half-space.

Keywords: Navier-Stokes equations; existence; regular solutions.

References

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