Quasiconvexity and the norms of the Beurling–Ahlfors transform

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Talk Abstract

The Beurling–Ahlfors transform $\mathcal{S}\colon L^p(\mathbb{C}) \to L^p(\mathbb{C})$ is a fundamental singular integral operator in the complex plane. A well-known conjecture due to Iwaniec asserts that

$$\|\mathcal{S}\|_{L^p(\mathbb{C}) \to L^p(\mathbb{C})} = \max\{p - 1, (p - 1)^{-1}\}.$$

This conjecture has a deep connection with Morrey's problem, which relates quasiconvexity and rank-one convexity in the vectorial Calculus of Variations. We will discuss recent progress in both problems by establishing quasiconvexity for a large family of geometric integrands. As applications, we derive sharp L^p estimates for the derivatives of quasiconformal mappings and sharp $L \log L$ estimates for the Jacobian determinant of an orientation-preserving mapping. Based on joint work with Kristensen [2] and Astala, Faraco, Kristensen and Koski [1].

Keywords: quasiconvexity, rank-one convexity, quasiconformal mappings.

References

[1] K. Astala, D. Faraco, A. Guerra, A. Koski, and J. Kristensen. In preparation.

[2] A. Guerra and J. Kristensen. Automatic Quasiconvexity of Homogeneous Isotropic Rank- One Convex Integrands. Archive for Rational Mechanics and Analysis, may 2022.