Yamabe systems, optimal partitions and nodal solutions to the Yamabe equation

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Talk Abstract

We give conditions for the existence of regular optimal partitions, with an arbitrary number $\ell \geq 2$ of components, for the Yamabe equation on a closed Riemannian manifold (M, g).

To this aim, we study a weakly coupled competitive elliptic system of ℓ equations, related to the Yamabe equation. We show that this system has a least energy solution with nontrivial components if $\dim M \geq 10$, (M, g) is not locally conformally flat and satisfies an additional geometric assumption whenever $\dim M = 10$. Moreover, we show that the limit profiles of the components of the solution separate spatially as the competition parameter goes to $-\infty$, giving rise to an optimal partition. We show that this partition exhausts the whole manifold, and we prove the regularity of both the interfaces and the limit profiles, together with a free boundary condition.

For $\ell = 2$ the optimal partition obtained yields a least energy sign-changing solution to the Yamabe equation with precisely two nodal domains.

Keywords: Competitive elliptic system, Riemannian manifold, critical nonlinearity, optimal partition, free boundary problem, regularity, Yamabe equation, sign-changing solution.

Acknowledgements

M. Clapp was partially supported by CONACYT (Mexico) through the grant for Project A1-S-10457. A. Pistoia was partially supported by Fondi di Ateneo "Sapienza" Università di Roma (Italy). H. Tavares was partially supported by the Portuguese government through FCT-Fundação para a Ciência e a Tecnologia, I.P., under the projects UID/MAT/04459/2020, PTDC/MAT-PUR/28686/2017 and PTDC/MAT-PUR/1788/2020.

References

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