Stability of a periodically perturbed point-vortex

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Talk Abstract

We present a result about the stability of a periodic Hamiltonian system in the plane with a singularity: a periodically perturbed point-vortex, [1]. In a perfect fluid, a point-vortex is essentially a singularity of the vorticity, and can be modeled by the Hamiltonian

$$H_0(x, y) = \frac{1}{2} ln(x^2 + y^2),$$

being x and y the usual rectangular coordinates in the plane. The associated system is integrable, with the particles rotating around the vortex in circular paths and the origin is trivially stable. We have studied this system after introducing an external periodic perturbation $p(t, x, y)$. The perturbed system models ideally the passive transport of particles in a perfect fluid under the action of a steady vortex placed at the origin and an external time-dependent background flow. We will see which hypothesis must be imposed on the perturbation $p(t, x, y)$ to preserve the stability of the origin. In this context, we apply the *Invariant Curve Theorem* in the analytical version presented in [2]. This allows to find a family of invariant curves by the Poincarp of our system. These curves surround the vortex and due to the low dimensionality, act as barriers to the solutions; therefore, the stability of the origin can be guaranteed. Recently, in [3] the authors proved a similar stability result under the action of a periodic background flow induced by a general polynomial field

$$\sum_{1 \leq i+j \leq N} a_{ij}(t) x^i y^j,$$

where a_{ij} are 2π-periodic continuous differentiable functions. The proof is obtained from a finite differentiable version of the *Invariant Curve Theorem* [4].

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Keywords: Vortex dynamics, KAM theory, particle advection, Mosers invariant curve theorem

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