

# Some Optimal Design Problems with Perimeter Penalisation

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## Talk Abstract

We obtain a measure representation for a functional arising in the context of optimal design problems under linear growth conditions (see Barroso, Matias and Zappale [1]). Starting from an energy  $F(\chi, u)$ , which has a bulk term depending on the symmetrised gradient of  $u$ , as well as a perimeter term, the functional in question is the relaxation of  $F(\cdot, \cdot)$  with respect to a pair  $(\chi, u)$ , where  $\chi$  is the characteristic function of a set of finite perimeter, corresponding to the optimal shape, and  $u$  is a function of bounded deformation. The perimeter term, which penalises the interface between the two regions  $\{\chi = 1\}$  and  $\{\chi = 0\}$ , is added to ensure compactness of minimising sequences.

In Barroso and Zappale [2] and [3], a similar investigation was undertaken in the case of non-standard p-q growth conditions on the original bulk energy densities, which now depend on the full gradient of the  $u$  variable, and where the energy also includes a perimeter penalisation term. In this setting, we showed in [2] that one of the relaxed functionals under consideration only admits a weak measure representation, whereas for the other a strong measure representation holds. Under some convexity assumptions, we provided a partial characterisation of the corresponding measures, a full representation was obtained in the one-dimensional setting.

In [3] we further identified some conditions under which the relaxation process gives rise to no concentration effects. In this case, we showed that the integral representation in question is composed of a term which is absolutely continuous with respect to the Lebesgue measure, and a perimeter term, but has no additional singular term.

**Keywords:** measure representation, relaxation, functions of bounded deformation, symmetric quasiconvexity, non-standard growth conditions

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## References

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