On the logistic equation for the fractional p-Laplacian

Antonio Iannizzotto[1], **Sunra Mosconi**[2], **Nikolaos S. Papageorgiou**[3]

[1] *University of Cagliari, Department of Mathematics and Computer Science, Italy*
[2] *University of Catania, Department of Mathematics and Computer Science, Italy*
[3] *National Technical University, Department of Mathematics, Athens, Greece*

Corresponding/Presenting author: antonio.iannizzotto@unica.it

Talk Abstract

We study the following Dirichlet type problem for a nonlinear nonlocal elliptic equation:

$$(P_\lambda) \qquad \begin{cases} (-\Delta)_p^s u = \lambda u^{q-1} - u^{r-1} & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $p \geq 2$, $s \in (0,1)$ s.t. $ps > N$, $q \in (1, p_s^*)$, $r \in (p, p_s^*)$, $\lambda > 0$ and the leading operator is the degenerate fractional p-Laplacian

$$(-\Delta)_p^s u(x) = 2 \lim_{\varepsilon \searrow 0} \int_{\{|x-y|>\varepsilon\}} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+ps}}\, dy.$$

The reaction is of logistic type with powers $q \in (1, p_s^*)$, $r \in (p, p_s^*)$, depending on a parameter $\lambda > 0$. Previous results on logistic equations with nonlocal operators are in [1, 4, 5]. We distinguish three cases:

(*a*) in the subdiffusive case $q < p$, for all $\lambda > 0$ problem (P_λ) has a unique solution u_λ with $u_\lambda \to 0$ as $\lambda \to 0^+$;

(*b*) in the equidiffusive case $q = p$, problem (P_λ) has no solution for $\lambda \leq \lambda_1$, and a unique solution u_λ for all $\lambda > \lambda_1$, with $u_\lambda \to 0$ as $\lambda \to \lambda_1^+$ ($\lambda_1 > 0$ is the principal eigenvalue of $(-\Delta)_p^s$ in Ω);

(*c*) in the superdiffusive case $q \in (p, r)$, there exists $\lambda_* > 0$ s.t. (P_λ) has no solution for $\lambda < \lambda_*$, (P_{λ_*}) has a solution u_*, and for all $\lambda > \lambda_*$ (P_λ) has two solutions $u_\lambda > v_\lambda$, with $u_\lambda \to u_*$ as $\lambda \to \lambda_*^+$.

In all cases, the map $\lambda \mapsto u$ is increasing. In cases (a), (b) we use the direct variational method (minimization) and a Brezis-Oswald approach for uniqueness, while in case (c) we find a second solution via truncations and the mountain pass theorem. Monotonicity results stem from a new strong minimum/comparison principle (see [2] for details). All results make use of the weighted Hölder regularity proved in [3].

Keywords: fractional p-Laplacian, logistic equation, comparison principle.

Acknowledgements

Work partially supported by PRIN programme and Fondazione di Sardegna.

References

[1] G. Carboni, D. Mugnai, On some fractional equations with convex-concave and logistic-type nonlinearities, *J. Differential Equations* **262** (2017) 2393–2413.

[2] A. Iannizzotto, S. Mosconi, N.S. Papageorgiou, On the logistic equation for the fractional p-Laplacian (submitted).

[3] A. Iannizzotto, S. Mosconi, M. Squassina, Fine boundary regularity for the degenerate fractional p-Laplacian, *J. Funct. Anal.* **279** (2020) art. 108659.

[4] E. Montefusco, B. Pellacci, G. Verzini, Fractional diffusion with Neumann boundary conditions: the logistic equation, *Discrete Contin. Dyn. Syst. Ser. B* **18** (2013) 2175–2202.

[5] A. Quaas, A. Xia, Existence and uniqueness of positive solutions for a class of logistic type elliptic equations in $\mathbb{R}^N$ involving fractional Laplacian, *Discrete Contin. Dyn. Syst.* **37** (2017) 2653–2668.