Unique continuation from the boundary for fractional problems

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Talk Abstract

I will present the recent results of paper [1], concerning the problem of unique continuation from boundary points for some fractional elliptic equations under outer homogeneous Dirichlet boundary conditions. I will describe a blow-up procedure which involves an Almgren type monotonicity formula and provides a classification of all possible homogeneity degrees of limiting entire profiles. The Caffarelli-Silvestre extension provides an equivalent formulation of the fractional equation as a local degenerate or singular problem in one dimension more, with mixed Dirichlet and Neumann boundary conditions. In the development of a monotonicity argument, the mixed boundary condition raises delicate regularity issues, which turn out to be quite difficult in dimension $N \geq 2$ due to the positive dimension of the junction set and some role played by the geometry of the domain. Such difficulties are overcome by a double approximation procedure: by approximating the potential with functions vanishing near the boundary and the Dirichlet N-dimensional region with smooth $(N+1)$-dimensional sets with straight vertical boundary, it is possible to construct a sequence of approximating solutions which enjoy enough regularity to derive Pohozaev type identities, needed to obtain Almgren type monotonicity formulas and consequently to perform blow-up analysis.

Keywords: fractional elliptic equations, unique continuation, monotonicity formula, boundary behavior of solutions.

References

[1] De Luca, A., Felli, V. and Vita, S., Strong unique continuation and local asymptotics at the boundary for fractional elliptic equations, *Advances in Mathematics*, to appear.