

Donati type theorems and compensated compactness in the periodic case

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Abstract

In 1890 Donati gave a classic result : a necessary condition on a strain tensor field that is orthogonal to all divergence free stress tensor fields. The condition is written in terms of second order derivatives of the strain tensor field, and for simply connected domains is equivalent to the so-called Saint Venant compatibility condition. 84 years later, in 1974, T.W. Ting (see [1]) obtained an extension of Donati's result : the necessary condition on a strain tensor field that is orthogonal to all divergence free stress tensor fields, is to be the symmetric gradient of some vector field belonging to a Sobolev space. In the last 20 years P. G. Ciarlet and his collaborators and students (see [2] and [3]) wrote several papers where extensions of the Donati's theorem were obtained for Dirichlet, Neumann and mixed boundary conditions. However, the periodicity condition was not considered and remained open until now. In the Homogenization Theory applied to Elasticity, a periodicity condition appears naturally when characterizing the homogenized elastic tensor in terms of the so called cellular problems, which are PDEs with periodic boundary conditions. In order to obtain the equivalence between the variational formulations in strain, in stress and in displacement, we are led naturally to a compensated compactness result stated by P. Suquet in 1987 (see [4]). The notion of compensated compactness has been introduced by L. Tartar in 1979 (see [5]) as a condition for the weak convergence of products between strain and stress fields. We will present three extensions of Donati's theorem in the periodic context, where the obtained conditions are necessary and sufficient.

Keywords: homogenization, homogenized elastic tensor, cellular problems.

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