Existence results for a turbulence $k-$epsilon model governing flows through permeable media

Hermengildo Borges de Oliveira[1,2]

[1]*FCT, Universidade do Algarve, Portugal*
[2]*CMAFcIO, Universidade de Lisboa, Portugal*

Corresponding/Presenting author: holivei@ualg.pt

Talk Abstract

In this talk, we consider turbulent flows through permeable media governed by the following general one-equation turbulence $k-$epsilon model,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \mathbf{div}\,(\nu_T(k)\mathbf{D}(\mathbf{u})) + \nabla p = \mathbf{g} - \mathbf{f}(\mathbf{u}) \ \text{ in } \ Q_T, \tag{1}$$

$$\operatorname{div}\mathbf{u} = 0 \ \text{ in } \ Q_T, \tag{2}$$

$$\frac{\partial k}{\partial t} + \mathbf{u}\cdot\nabla k - \operatorname{div}(\nu_D(k)\nabla k) = \nu_T(k)|\mathbf{D}(\mathbf{u})|^2 + P(\mathbf{u},k) - \varepsilon(k) \ \text{ in } \ Q_T, \tag{3}$$

$$\mathbf{u} = \mathbf{u}_0 \quad \text{and} \quad k = k_0 \quad \text{in } \ \Omega\times\{0\}, \tag{4}$$

$$\mathbf{u} = \mathbf{0} \quad \text{and} \quad k = 0 \quad \text{on } \ \Gamma_T, \tag{5}$$

set in a cylinder $Q_T := \Omega\times(0,T)$, where $\Gamma_T := \partial\Omega\times(0,T)$, $\Omega\subset\mathbb{R}^d$, $d\geq 2$, is a bounded domain, with its boundary denoted by $\partial\Omega$, and T is a given positive constant. The velocity field $\mathbf{u}$ and the pressure p are, in fact, averages that result by the application of the two averaging concepts. The averaged tensor $\mathbf{D}(\mathbf{u})$ is the symmetric part of the averaged gradient $\nabla\mathbf{u}$, k is the turbulent kinetic energy, ν_T is the turbulent, or eddy, viscosity, ν_D is the turbulent diffusion and ε is the dissipation of turbulent kinetic energy. For the sake of simplicity, the medium porosity is assumed to be constant. The feedback term $\mathbf{f}(\mathbf{u})$ accounts for the resistance made by the rigid matrix of the permeable medium to the flow, whereas $\mathbf{g}$ stands for a body forces field. The function $P(\mathbf{u},k)$ appears as an output of the averaging process, and it is a production term of turbulent kinetic energy that accounts for the solids inside the fluid. In this talk, we study the existence of weak solutions to both the steady-state and the transient version of the problem (1)-(5), under distinct conditions on the growth of $\mathbf{f}(\mathbf{u})$, $\nu_T(k)$, $\nu_D(k)$, $\varepsilon(k)$ and $P(\mathbf{u},k)$. This talk is based in the papers [1, 2] and in some recent work [3] by the author.

Keywords: fluid flows through permeable media, turbulence $k-$epsilon model, existence of weak solutions.

Acknowledgements

This work is partially supported by Fundação para a Ciência e a Tecnologia, Portugal, through the project UIDB/04561/2020.

References

[1] H.B. de Oliveira and A. Paiva. Existence for a one-equation turbulent model with strong nonlinearities. *J. Elliptic Parabol. Equ.* **3** (2017), no. 1-2, 65–91.

[2] H.B. de Oliveira and A. Paiva. A stationary one-equation turbulent model with applications in porous media. *J. Math. Fluid Mech.* **20** (2018), no. 2, 263–287.

[3] H.B. de Oliveira. Parabolic turbulent $k-$epsilon model with applications in fluid flows through permeable media. To appear.