

On the minimization of general energies with attractive-repulsive behaviour

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Talk Abstract

The celebrated “liquid drop model” by Gamow is one of the oldest and most studied energies of attractive-repulsive type, and it has gathered a huge interest among physicists and mathematicians. In the last years several generalisations of the model have been studied (see for instance [1, 2, 3]), and now many important properties are known, though still some fundamental questions are open, even in the original model. In addition, people have started to consider the minimization in the class of L^1 positive functions, instead than in the class of sets. In this talk, we will describe the main features of the problem, and we will concentrate ourselves in the even more general case of minimization among positive measures, already considered by some authors but largely open. We will briefly present some properties, proven very recently, and some open questions (see [4, 5]). Some of the results have been proved in collaboration with Carazzato, Fusco, Novaga.

Keywords: non-local energies, charged droplets, regularity of minimizers.

References

- [1] Burchard, A., Choksi, R. and Topaloglu, I., Nonlocal shape optimization via interactions of attractive and repulsive potentials, *Indiana Univ. Math. J.*, 67 (1), 2018, pp. 375–395.
- [2] Frank, R. and Lieb, E., Proof of spherical flocking based on quantitative rearrangement inequalities, *Ann. Sc. Norm. Super. Pisa Cl. Sci.*, 22 (3), (2021), pp. 1241–1263.
- [3] Novaga, M. and Pratelli, A., Minimisers of a general Riesz-type problem, *Nonlinear Anal.*, 209, 2021, Paper No. 112346, 27 pp.
- [4] Carazzato, D., Fusco, N. and Pratelli, A., Minimality of balls in the small volume regime for a general Gamow type functional, to appear on *Adv. Calc. Var.*, 2022.

- [5] Carazzato, D. and Pratelli, A., On the existence and boundedness of minimizing measures for a general form of non-local energies, *preprint* (2022). Available at <https://arxiv.org/abs/2205.09412>.