

How to recognize constant functions through double integrals of their difference quotients

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Talk Abstract

We present some results contained in [1] that address a classical open question by H. Brezis [2] and R. Ignat [3] concerning the characterization of constant functions through double integrals that involve their difference quotients.

On the negative side, we present a counterexample that shows that the natural necessary conditions introduced in [3] are not sufficient without some kind of summability assumption. On the positive side, we show that the answer to the question is positive if we restrict either to functions that are bounded and approximately differentiable almost everywhere, or to functions with bounded variation.

We also discuss the cases that remain open, and how they motivate further questions in measure theory.

Keywords: difference quotients, nonlocal functionals, bounded variation functions, approximate differentiability.

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References

- [1] M. GOBBINO, N. PICENNI. On the characterization of constant functions through nonlocal functionals. *Commun. Contemp. Math.*, to appear.
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